South Carolina College- and Career-Ready Standards for Mathematics

8th Grade Support Document

South Carolina Department of Education
Office of Standards and Learning
June 2016 - DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Grade 8 Overview

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for middle school into *Course Coversheets* and *Units*.

- Each middle school *Course Coversheet* organizes the middle school course standards into possible instructional units and provides links to specific middle school course *Units*.
- Each middle school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Potential instructional strategies and lessons
  - Resources for the unit
  - Sample formative assessment tasks and questions

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Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
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Grade 8 Coversheet

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Unit Title
Transformational Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.GM.1 Investigate the properties of rigid transformations (rotations, reflections, translations) using a variety of tools (e.g., grid paper, reflective devices, graphing paper, technology). a. Verify that lines are mapped to lines, including parallel lines. b. Verify that corresponding angles are congruent. c. Verify that corresponding line segments are congruent. ○ Understand that rigid transformations maintain congruence (rotations, reflections, translations) ○ A line segment can be rotated, reflected, and translated and remain the same length. ○ Angles can be rotated, reflected, and translated and still have the same measure. ○ Parallel lines can be rotated, reflected, and translated and still remain parallel. **8.GM.2** Apply the properties of rigid transformations (rotations, reflections, translations). a. Rotate geometric figures 90, 180, and 270 degrees, both clockwise and counterclockwise, about the origin. b. Reflect geometric figures with respect to the *x*-axis and/or *y*-axis. c. Translate geometric figures vertically and/or horizontally. d. Recognize that two-dimensional figures are only congruent if a series of rigid transformations can be performed to map the pre-image to the image. e. Given two congruent figures, describe the series of rigid transformations that justifies this congruence. ○ Recognize that the pre-image is the original figure, and the image is the figure after transformations have been applied. ○ Two-dimensional figures are congruent if corresponding sides and angles are equal. ○ A two-dimensional figure is congruent to another if the second one can be obtained from the first by a sequence of rotations, reflections, or translations. ○ When given two congruent figures, they can describe a sequence that exhibits the congruence between them.

8.GM.3 Investigate the properties of transformations (rotations, reflections, translations, dilations) using a variety of tools (e.g., grid paper, reflective devices, graphing paper, dynamic software).

a. Use coordinate geometry to describe the effect of transformations on two-dimensional figures.
b. Relate scale drawings to dilations of geometric figures.
- Understand that a dilation is not a rigid transformation.
- Understand that a dilation with a scale factor between 0 and 1 results in a shrink/reduction. Limit scale factors to positive rational numbers.
- Understand that a dilation with a scale factor greater than 1 results in a stretch/enlargement.
- Recognize that the only dilation that maintains congruence is with a scale factor of 1.
- Describe the effect of rotations, reflections, translations, and dilations on two-dimensional figures using coordinates.

8.GM.4 Apply the properties of transformations (rotations, reflections, translations, dilations).

a. Dilate geometric figures using scale factors that are positive rational numbers.
b. Recognize that two-dimensional figures are only similar if a series of transformations can be performed to map the pre-image to the image.
c. Given two similar figures, describe the series of transformations that justifies this similarity.
d. Use proportional reasoning to find the missing side lengths of two similar figures.
- Two-dimensional figures are similar if corresponding sides are proportional and corresponding angles are congruent.
- A two-dimensional figure is similar to another if the second one can be obtained from the first by a sequence of rotations, reflections, translations, and/or dilations.
- When given two similar figures, describe a sequence that exhibits the similarity between them.

8.GM.5 Extend and apply previous knowledge of angles to properties of triangles, similar figures, and parallel lines cut by a transversal.

a. Discover that the sum of the three angles in a triangle is 180 degrees.
b. Discover and use the relationship between interior and exterior angles of a triangle.
c. Identify congruent and supplementary pairs of angles when two parallel lines are cut by a transversal.
d. Recognize that two similar figures have congruent corresponding angles.
- Given two interior angle measurements for any triangle, you can find all interior and exterior angle measurements for that triangle.
- The sum of the measures of the three angles in any triangle is 180 degrees.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles.
- A transversal is the line that cuts two parallel lines.
- Algebraic expressions should be used in addition to numerical values to represent angle measures.
- Recognize that alternate interior, alternate exterior, and corresponding angles are congruent when two parallel lines are cut by a transversal. (This does not hold true for non-parallel lines.)
- Recognize that consecutive interior angles are supplementary when two parallel lines are cut by a transversal. (This does not hold true for non-parallel lines.)
- When two angles of one triangle are congruent to two angles of another, the triangles are similar.

New Academic Vocabulary for This Unit
• Pre-image • Rigid Transformations/Rigid Motions • Isometry • Clockwise • Counterclockwise • Dilations • Center of Dilation • Transversal • Exterior Angles • Corresponding Angles • Alternate Interior Angles • Alternate Exterior Angles • Consecutive Interior Angles

Prior Knowledge Required for this Unit
• Plot points in all four quadrants of the coordinate plane (6.NS.8) • Graph proportional relationships on the coordinate plane (7.RP.2) • Determine when two quantities are in a proportional relationship (7.RP.2) • Construct triangles and quadrilaterals given specific measures/parameters of either angles or sides (7.GM.2) • Write equations to solve problems involving the relationships between angles formed by two intersecting lines, including supplementary, complementary, vertical, and adjacent (7.GM.5)
Subsequent Knowledge Related to this Unit
By the end of this unit, students should be fluent in plotting points in the coordinate plane. In high school mathematics courses, students will extend their knowledge of transformations to vectors and matrices. They will also develop proofs for the angle relationships formed by lines (parallel and nonparallel) cut by a transversal. Additionally, high school courses will extend students' knowledge of dilations to include negative scale factors and create fractals. Proofs for congruence and similarity (e.g., Side-Angle-Side, Side-Side-Side, Angle-Angle) will also draw on students' knowledge of transformations.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop an understanding of the impact transformations have on congruence and similarity among figures.

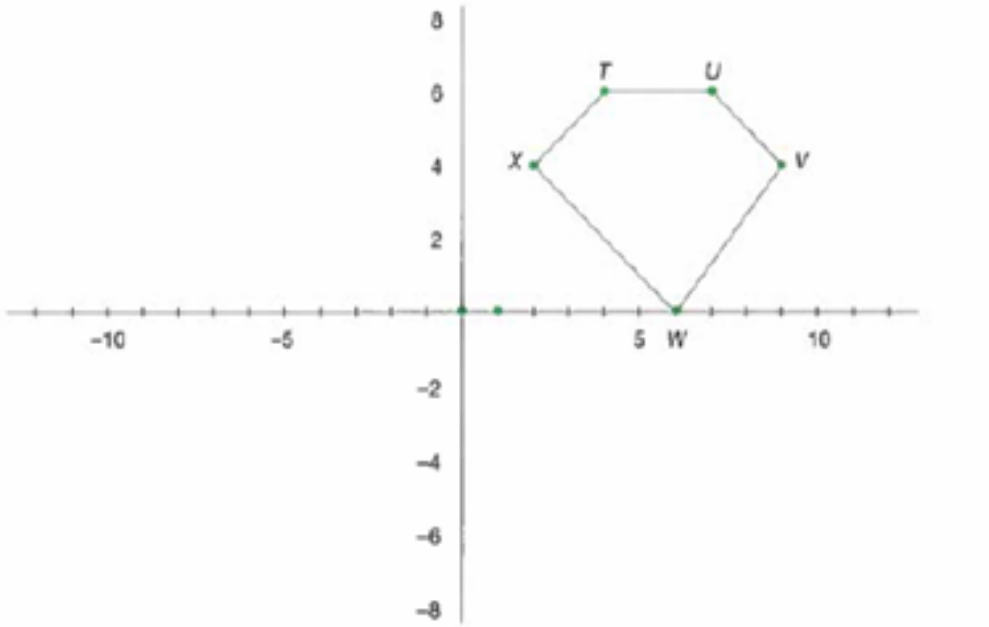
Potential Instructional Strategies/Lessons

• Use patty paper, transparency sheets, or MIRAs to show transformations
• Provide sentence starters for students to be able to describe the effects of transformations
• Use paper folding to discover/prove that the sum of the angles in a triangle equal 180^0
• Geogebra software (http://www.geogebra.org/)
  ○ Use geometry software to make and compare transformations
  ○ Use geometry software to show two parallel lines cut by a transversal
• Geometer's Sketchpad (http://www.dynamicgeometry.com/)
  ○ Use geometry software to make and compare transformations
  ○ Use geometry software to show two parallel lines cut by a transversal
• Compass and straightedge

Resources
8.GM.1 - This website allows students to investigate rigid transformations on a coordinate plane. http://www.sciencekids.co.nz/gamesactivities/math/transformation.html 8.GM.2 - This website contains a lesson/activity that allows students to strengthen their skills with rigid transformations. Isometry is the term used in this lesson. http://www.cpalms.org/Public/PreviewResourceLesson/Preview/64769 8.GM.4 - This website provides students with the opportunity to interactively work with dilations. http://nlvm.usu.edu/en/nav/frames_asid_295_g_3_t_3.html?open=activities&from=topic_t_3.html 8.GM.5 - This website allows students to manipulate pairs of lines cut by a transversal and further explore any pair relationships. http://www.mathwarehouse.com/geometry/angle/parallel-lines-cut-transversal.php

Sample Formative Assessment Tasks/Questions
8.GM.1 and 8.GM.2: This task challenges a student to use transformations, reflections and rotations on a coordinate grid.  Aaron's Designs.pdf Source: Mathematics Assessment Resource Service
8.GM.3 and 8.GM.4: In this task, students sketch a logo on graph paper use coordinate points to perform transformations.  Corporate Logos Project.docx

8.GM.3 and 8.GM.4: Which of the following represents the coordinates of the vertices after a rotation of 180° about the origin?

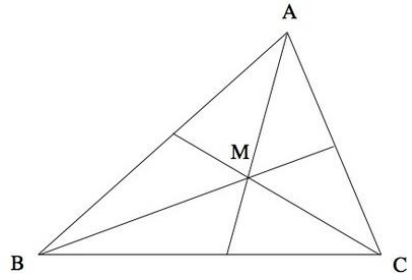


a. *T*'(-4, -6), *U*'(-7, -6), *V*'(-9, -4), *W*'(-6, 0), *X*'(-2, -4)
b. *T*'(-4, 6), *U*'(-7, 6), *V*'(-9, 4), *W*'(-6, 0), *X*'(-2, 4)
c. *T*'(4, -6), *U*'(7, -6), *V*'(9, -4), *W*'(6, 0), *X*'(2, -4)
d. *T*'(6, 4), *U*'(6, 7), *V*'(4, 9), *W*'(0, 6), *X*'(4, 2)

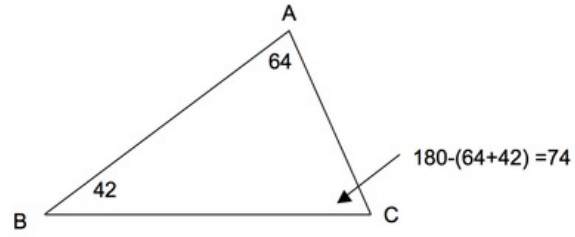
Answer: a. *T*'(-4, -6), *U*'(-7, -6), *V*'(-9, -4), *W*'(-6, 0), *X*'(-2, -4)

Source: Mathematics Assessment Sampler 6-8

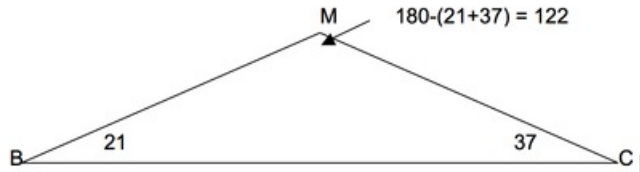
8.GM.5: In triangle Δ*ABC*, point *M* is the point of intersection of the bisectors of angles $\angle BAC$, $\angle ABC$, and $\angle ACB$. The measure of $\angle ABC$ is 42°, and the measure of $\angle BAC$ is 64°. What is the measure of $\angle BMC$?



Answer: All angle measurements are in degrees.
The solution is obtained by applying the Triangle Sum Theorem twice. First apply it to the Δ*ABC* to find the measure of $\angle ACB$. This angle has measure 180°−(64°+42°) = 180°−(106°) =74°:



Now consider the triangle BMC. Since the segment BM bisects the angle ABC of the triangle, we have the measure of $\angle MBC$ is half the measure of $\angle ABC$, which is half of 42°, or 21°. Similarly, the measure of $\angle MCB$ is half of angle $\angle ACB$, which is half of 74°,which is 37°. Now use the Triangle Sum Theorem on the Δ*BMC* to find that the measure of $\angle BMC$ is 180°−(37°+21°)=180°−58°=122°:



Source: Illustrative Mathematics

Unit Title
Exponents
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.NS.1 Explore the real number system and its appropriate usage in real-world situations. a. Recognize the differences between rational and irrational numbers. b. Understand that all real numbers have a decimal expansion. c. Model the hierarchy of the real number system, including natural, whole, integer, rational, and irrational numbers. ○ Rational numbers are all fractions, decimals (terminating and repeating), and integers ○ Irrational numbers are non-terminating and non-repeating decimals ○ Irrational numbers include the square roots of non-perfect squares **8.NS.2** Estimate and compare the value of irrational numbers by plotting them on a number line. ○ Use square roots of perfect squares as benchmarks on a number line **8.NS.3** Extend prior knowledge to translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Include the conversion of repeating decimal numbers to fractions. ○ Use algorithmic approach to teach all repeating decimals **8.EEI.1** Understand and apply the laws of exponents (i.e. product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property, negative exponents) to simplify numerical expressions that include integer exponents. ○ Use conceptual understanding to demonstrate mastery of the laws ○ Negative exponents indicates a fractional value not the additive inverse of the base raised to the positive exponent (ex: $3^{-2} \neq -9$) ○ Recognize difference between opposite of a value squared and a negative value squared (ex: $-3^2 \neq (-3)^2$) **8.EEI.2** Investigate concepts of square and cube roots. a. Find the exact and approximate solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number. b. Evaluate square roots of perfect squares. c. Evaluate cube roots of perfect cubes. d. Recognize that square roots of non-perfect squares are irrational. ○ A number being multiplied by itself twice as a perfect square ○ A number being multiplied by itself three times as a perfect cube ○ Inverse operations of squaring and cubing a number ○ Use square roots of perfect square to approximate square roots of non-perfect squares

8.EEI.3 Explore the relationship between quantities in decimal and scientific notation.

a. Express very large and very small quantities in scientific notation in the form $a \times 10^b = p$ where $1 \leq a < 10$ and b is an integer.

b. Translate between decimal notation and scientific notation.

c. Estimate and compare the relative size of two quantities in scientific notation.

- Emphasize that scientific notation must be a rational number greater than or equal to 1 but less than 10

8.EEI.4 Apply the concepts of decimal and scientific notation to solve real-world and mathematical problems.

a. Multiply and divide numbers expressed in both decimal and scientific notation.

b. Select appropriate units of measure when representing answers in scientific notation.

c. Translate how different technological devices display numbers in scientific notation.

- Recognize that when multiplying and dividing numbers expressed in scientific notation the order of factors will not matter
- Recognize that the answer when performing operations with scientific notation will not necessarily be in scientific notation which leads to conversion back to scientific notation

8.EEI.7 Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations.

a. Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides.

b. Recognize the three types of solutions to linear equations: one solution($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$).

- Students solve one-variable equations with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms.
- Recognize that infinitely many solutions means that any value substituted for the variable will make the equality true
- Recognize that no solution means that no value of the variable will make the equality true

8.DSP.5 Organize data in matrices with rational numbers and apply to real-world and mathematical situations.

a. Understand that a matrix is a way to organize data.

b. Recognize that a $m \times n$ matrix has m rows and n columns.

c. Add and subtract matrices of the same size.

d. Multiply a matrix by a scalar.

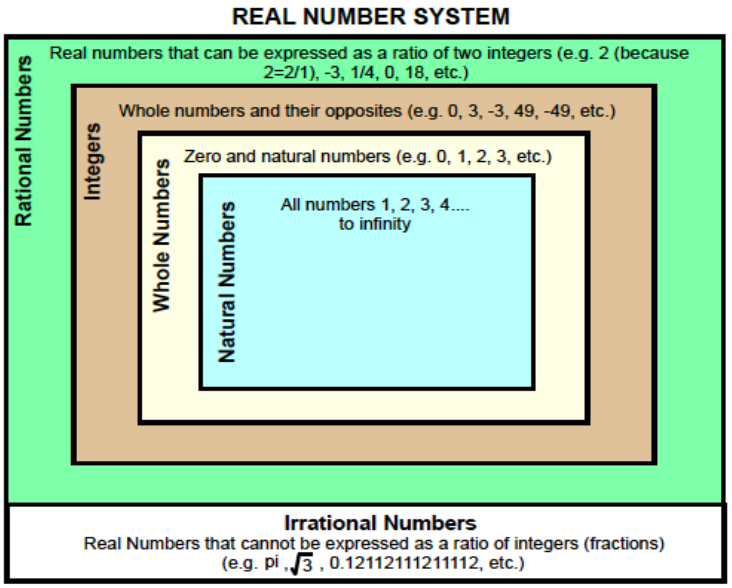
- When naming a matrix, rows always precede columns
- When performing operations with matrices, use corresponding cells/entries
- Addition and subtraction of matrices must be performed on matrices with like rows and columns
- Scalar multiplication for a matrix is representative of the distributive property of an expression

New Academic Vocabulary for This Unit
• Square root • Perfect square • Cube root • Perfect cube • Radical • Irrational number (π) • Scientific notation • Matrix • Rows • Columns • Scalar • Subset

Prior Knowledge Required for this Unit
• Fluent with the subsets of the number systems (real, natural, whole, integers, and rational) (6.NS.6) • Fluent with plotting points on a number line (horizontal and vertical) (6.NS.8) • Convert rational numbers to decimals using long division (terminating and repeating) (6.NS.9, 7.NS.5) • Understand exponents as repeated multiplication (5.NSBT.2) • Compute fluently with integers (7.NS.1, 7.NS.2) • Translate among multiple representations of rational numbers, including repeating decimals to fractions (6.NS.9, 7.NS.5) • Understand the implication of inverse operations (6.EEI.7) • Fluent with the powers of ten to make a number larger or smaller in the place value system (5.NSBT.2) • Fluent in solving multi-step linear equations and inequalities with the variable on one side and rational number coefficients. Include combining like terms and applying the distributive property (7.EEI.4) • Knowledge of multiple ways to display data (3.MDA.3, 3.MDA.4, 4.MDA.4, 6.DS.4)
Subsequent Knowledge Related to this Unit
Students will extend their knowledge of the laws of exponents to algebraic expressions and rational exponents for high school courses. The students' fluency in solving multi-step equations with variables on both sides will assist them with solving systems of equations later in Grade 8 and systems of inequalities in high school courses. Students will extend their knowledge of matrices in high school courses.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop the computational skills needed for work within the real number system including solving multi-step linear equations and inequalities and simplifying expressions which include the use of integer exponents.

Potential Instructional Strategies/Lessons

- Algebra Tiles
- Human Number Line
- Graphic Organizers



- The colored section of this chart can be thought of as stacking on top of each other like Russian stacking dolls with natural numbers (sometimes called counting numbers) being the smallest "doll". Whole numbers include all natural numbers, but also incorporate zero. Integers include all natural and whole numbers and their opposites. Finally, rational numbers contain all natural and whole numbers and their opposites along with fractions and decimals. They can be written in fractional form. Irrational numbers are also part of the Real Number System. However, they cannot be written as a fraction.

- Algebraic Proof for Negative Exponent: http://www.projectstar-edex.com/watch/?v=AUr6ZaAlyqo
- Technology with Scientific Notation: http://mathbits.com/MathBits/TISection/General/ScientificNotation.htm

Resources
6–8 Progression Document for Expressions and Equations: https://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf This document supports instruction of standards associated with Expressions, Equations, and Inequalities https://www.engageny.org/sites/default/files/resource/attachments/math-g8-m4-teacher-materials.pdf. 8.NS.1 - Distinguish between rational and irrational numbers. https://learnzillion.com/resources/8892 8.EEI.1 - This website demonstrates how to derive the laws of exponents. Use this website as a guide for lesson planning. http://www.mathsisfun.com/algebra/exponent-laws.html 8.EEI.1 - Why does $x^0 = 1$? http://www.homeschoolmath.net/teaching/negative_zero_exponents.php 8.EEI.2 - This document is an activity for learning squares and square roots. http://betterlesson.com/lesson/resource/1987846/perfect-squares-tile-activity-pdf 8.EEI.2 - This document is an activity for learning cubes and cube roots. http://betterlesson.com/lesson/resource/2599382/perfect-cubes-activity-pdf?from=section_resources_title 8.EEI.3 - Relationship between decimal notation and scientific notation. http://www.regentsprep.org/regents/math/algebra/AO2/TScicard2.htm 8.EEI.4 - Applying concepts of decimal and scientific notation. https://ims.ode.state.oh.us/ODE/IMS/Lessons/Content/CMA_LP_S01_BI_L08_I08_01.pdf 8.EEI.7 - Solving linear equation and inequalities with variables on both sides. http://www.doe.virginia.gov/testing/solsearch/sol/math/8/mess_8-15a.pdf 8.DSP.5 - This blog provides examples and instructional techniques for operations with matrices. http://www.shelovesmath.com/algebra/advanced-algebra/matrices-and-solving-systems-with-matrices/

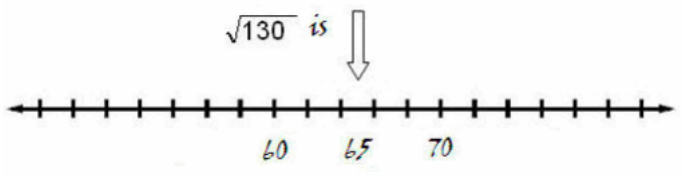
Sample Formative Assessment Tasks/Questions

8.NS.1: Analyze Robin's reasoning in her answer to the test question about rational or irrational numbers. Does she have a deep understanding of rational and irrational numbers? Does her reasoning make sense? If not, what misconceptions does she have about this topic? Create a study guide with explanations, examples, and graphics to help clear up any misconceptions students might have over these topics.

3rd Block Robin Radical

1. Is $\sqrt{130}$ rational or irrational? Where would $\sqrt{130}$ be located on a number line? Explain your reasoning.

$\sqrt{130}$ is a rational number because 130 is even. All rational numbers are even and irrational numbers are odd when the numbers are under the square root sign. The square root sign is the opposite of squaring a number. Squaring a number is the same as raising it to the 2nd power. So, to find the value of a number under the square root sign, you divide it by 2. So, 130÷2 is 65. The answer is 65.



Source: GA Department of Education (See page 18)

8.NS.2: Without using your calculator, label approximate locations for the following numbers on a number line.
a. π
b. -(½ x π)
c. $2\sqrt{2}$
d. $\sqrt{17}$

a. greater than 3.
b. $-(\frac{1}{2} \times \pi)$ is slightly less than −1.5.
c. $(2\sqrt{2})^2 = 4\sqrt{2} = 8$ and $3^2 = 9$, so $2\sqrt{2}$ is slightly less than 3.
d. $\sqrt{16} = 4$, so $\sqrt{17}$ is slightly greater than 4.

(Illustrative Mathematics)

8.EEI.1: This task will help students develop the meaning of negative integer exponents.

Extending the Definitions of Exponen Source: Illustrative Mathematics

8.EEI.2: Ashley and Brandon have different methods for finding square roots. Ashley's Method: To find the square root of *x*, find a number so that the product of the number and itself is *x*. For example, 2 x 2 = 4 so the square root of 4 is 2. Brandon's Method: To find the square root of *x* by ½. For example 4 x ½ = 2 so the square root of 4 is 2. Whose method is NOT correct? Explain why the method you chose is not correct.

Answer: Brandon's method is not correct. His method only works for the square root of 4. It would not work for the square root of 36. Half of 36 is 18, but the square root of 36 is 6 since 6 x 6 = 36. Ashley describes the correct way to find square roots.

8.EEI.3: The estimated area of a field is 9×10^4 square yards and the estimated area of a second field is 4.5×10^3 square yards.

A. How many times greater is the larger field than the smaller field?
B. How many square feet is each field (write in scientific notation)?
C. If you compare the two fields in square feet, how many times greater is the larger field than the smaller field?

Answer:
Part A: Students can convert the areas from scientific notation to standard form. In doing so, they see the first field has an area of 90,000 square yards, and the second has an area of 4,500 square yards. By establishing a ratio between these two values, students can see the first field's square yardage is 20 times larger.
Part B: There are 9 square feet in a square yard. Using this information, students can scale up by a multiplier of 9 to see that there are 810,000 square feet in the first field and 40,500 square feet in the second.
Part C: The ratio of 810,000 square feet to 40,500 square feet simplifies to 20 to 1. Students should recognize this answer should match that of Part A. Though the unit of measurement being used was changed from square yards to square feet, the actual areas of both fields remains constant.

8.EEI.4: In this lesson, students make conjectures to discover how to multiply and divide numbers written in scientific notation.


National Debt.pdf

Source: Ohio Department of Education

8.EEI.7a: Solve for x: $9(3 - 2x) = 2(10 - 8x)$

Answer: $x = 3.5$ The first step to solve this equation would be to perform the distributive property to get an equivalent equation $27 - 18x = 20 - 16x$. The next step is to combine like terms by transferring them to opposite sides of the equal sign by performing the inverse operation $7 = 2x$. The next step would be to divide each side of the equation by 2 to get your solution $x = 3.5$. The final step is to make sure your solution to your equation is true by substituting your value of x into the original equation and solve.

8.EEI.7b: Identify the type of solution you would expect from the following equation: $8(x + 4) - 7 = 13x + 25 - 5x$. Solve the problem algebraically to assess your hypothesis.

Answer: Infinitely many solutions - Students should apply the distributive property to the left side of the equation to attain $8x + 32 - 7$. By combining the like terms on each side of the equation, $8x + 25 = 8x + 25$ is attained. Students should recognize that the two sides are equivalent; if not, they should collect variables and integers on either side of the equation to get $0 = 0$. When both sides of the equation yield the same value, the result is infinitely many solutions.

8.DSP.5: Tae owns a gaming company that sells three different types of games: virtual, board, and card. He has two stores in South Carolina. The tables below display his company's sales for the two months.

STORE ONE	Virtual	Board	Card
August	152	47	63
December	219	92	81

STORE TWO	Virtual	Board	Card
August	114	81	91
December	187	120	73

Construct a matrix that displays the average sales of the two stores for August and December.

Answer: Students should add values in the two matrices. Next, they should multiply a by a scalar of ½ which would be equivalent to dividing the values by 2.

Average	Virtual	Board	Card
August	133	64	77
December	203	106	77

Unit Title
Algebraic Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.GM.6 Use models to demonstrate a proof of the Pythagorean Theorem and its converse. ○ Apply previous knowledge of area of squares to model the Pythagorean Theorem. ○ Use the Pythagorean Theorem to find the missing side of a right triangle. ○ Identify the parts of a right triangle (legs and hypotenuse) ○ Recognize the diagonal of a parallelogram with right angles as the hypotenuse of the right triangles formed ○ Determine if a triangle is a right triangle by using the Pythagorean Theorem ○ Verify the Pythagorean Theorem by examining the area of squares coming off of each side of the right triangle ○ Identify Pythagorean triples ○ Explain a proof of the Pythagorean Theorem **8.GM.7** Apply the Pythagorean Theorem to model and solve real-world and mathematical problems in two and three dimensions involving right triangles. ○ Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two dimensions ○ Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in three dimensions ○ Applying the Pythagorean Theorem to a rectangular prism will require students to determine the diagonal of one of the bases and use the determined length to serve as a leg for the triangle whose hypotenuse runs through the figure.

8.GM.8 Find the distance between any two points in the coordinate plane using the Pythagorean Theorem.
- Create triangles on the coordinate plane to find the diagonal distance between two points.
- The diagonal distance will be equivalent to the length of the triangle's hypotenuse.
- Use the Pythagorean Theorem (instead of the distance formula) to find the distance between two points in a coordinate plane
- Construct a right triangle on a coordinate plane to determine the distance between two points
- Determine the length of the diagonal or hypotenuse of a right triangle on a coordinate plane
- Use the coordinate plane to create a right triangle relationship whereby the distance between two points can be determined by solving for the hypotenuse using the Pythagorean Theorem.

8.GM.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres and the surface area of cylinders.
- Recognize that the volume of a cone will be $\frac{1}{3}$of the volume of a cylinder with an equivalent height.
- Recognize that "length x width x height" will only work for determining the volume of a rectangular prism and not be a viable expression for determining the volume of a sphere, cylinder, or cone.
- Use appropriate formulas for volume of cones, cylinders, and spheres and the surface area of cylinders in real-world and mathematical situations

8.EEI.2 Investigate concepts of square and cube roots.

a. Find the exact and approximate solutions to equations of the form $x^2 = p \; and \; x^3 = p$ where p is a positive rational number.

b. Evaluate square roots of perfect squares.

c. Evaluate cube roots of perfect cubes.

d. Recognize that square roots of non-perfect squares are irrational.
- Recognize $\sqrt{8}$ as an exact value and 2.828 as an approximate value.
- Recognize taking the square root as the inverse operation to squaring a number and taking the cube root as the inverse operation of cubing a number.
- A number being multiplied by itself twice as a perfect square
- A number being multiplied by itself three times as a perfect cube
- Inverse operations of squaring and cubing a number
- Use square roots of perfect square to approximate square roots of non-perfect squares

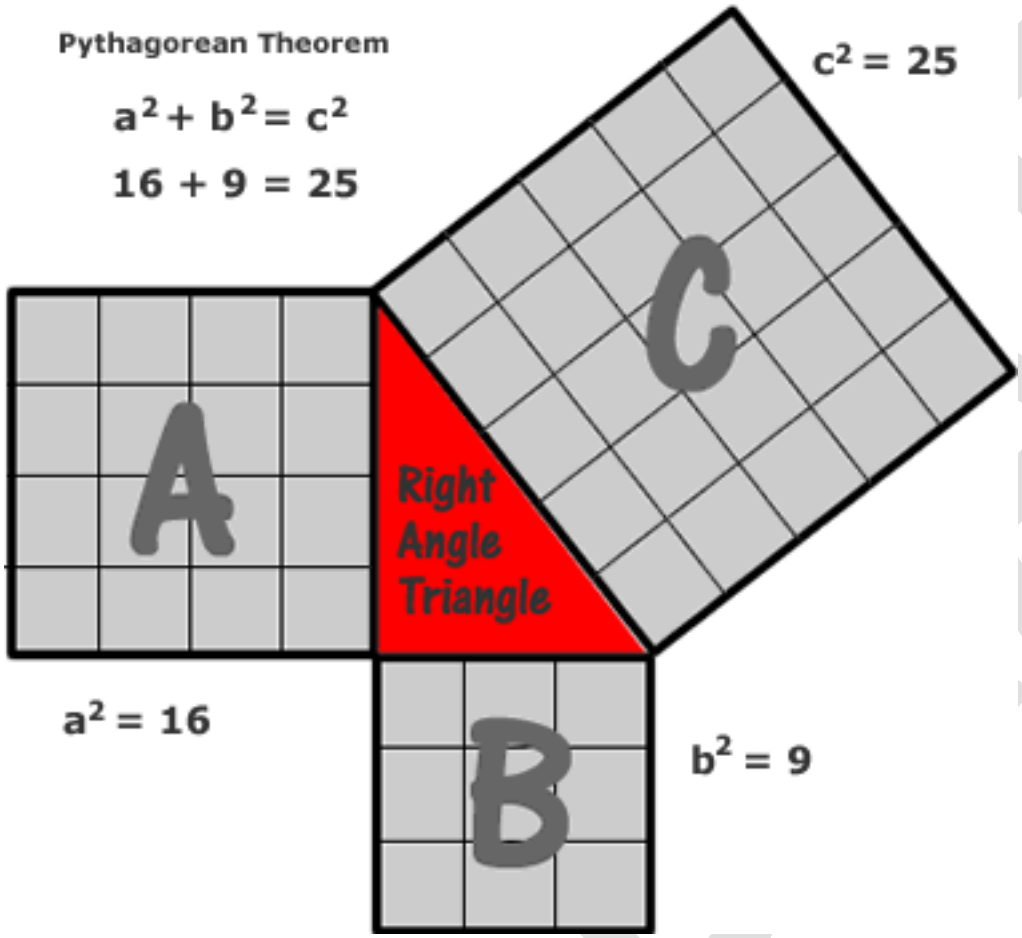
New Academic Vocabulary for This Unit
• Proof • Pythagorean Theorem • Converse • leg • hypotenuse

Prior Knowledge Required for this Unit
• Distance on a coordinate plane where the *x* or *y* coordinates are the same (6.NS.8) • Evaluating expressions involving whole number exponents (6.EEI.1) • Solve one-step linear equations for a given unknown (6.EEI.7) • Area and volume of triangles, special quadrilaterals, and right rectangular prisms (6.GM.1) • Evaluating expressions with positive rational numbers using the fraction bar as a grouping symbol (7.EEI.3) • Laws of Exponents with numerical values (7.EEI.5) • Construct special quadrilaterals with given parameters (7.GM.2) • Volume and surface area of three-dimensional shapes by decomposing shapes into cubes, rectangular prisms, and triangular prisms to derive formulas for volume and surface area (7.GM.6) • Concepts of perfect squares and perfect cubes as well as square roots and cube roots (8.EEI.2)
Subsequent Knowledge Related to this Unit
In high school, students will use their knowledge of the Pythagorean Theorem to derive the Distance Formula. Additionally, students will begin to determine the missing side lengths for non-right triangles using the Laws of Sines and Cosines. Knowledge of square and cube roots will be extended to develop an understanding of exponential and logarithmic functions in high school. These skills will also be used to build a conceptual understanding of imaginary and complex numbers.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop the computational skills needed to work with measurements and distances associated with two- and three-dimensional figures.

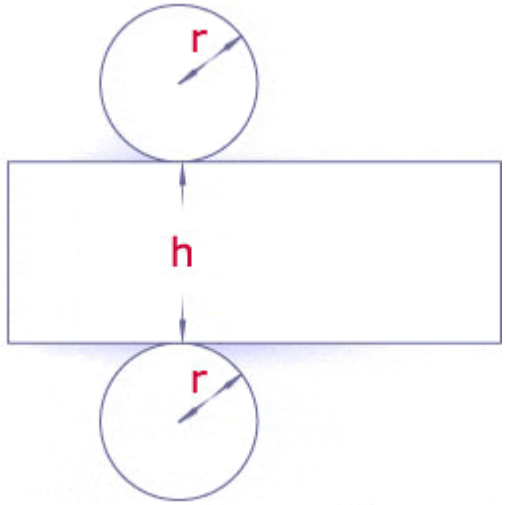
Potential Instructional Strategies/Lessons

- **Visual Models**

Using the prior knowledge of area of squares, students can prove that the area of square C is equal to the sum of the areas of squares A and B.



- **Nets**



Students should understand that a cylinder is a combination of circles and a rectangle. The two circles make up the base and the rectangle is "rolled up" to make the sides for the cylinder. Have the students roll a piece of 8 ½ x 11 paper and with a separate sheet of paper cut two circles that are the same size as the ends of the rolled up sheet. This allows them to see the construction and understand that the length of the rectangle is the same as the height of the cylinder and the width of the rectangle is the circumference of the circles at the end. (Consider having students use pre-cut string to identify measurements when making these constructions.)

Resources
Perigal's Dissection - Pythagorean Theorem (Beginning on page 24) http://centraledesmaths.uregina.ca/RR/database/RR.09.97/bracken1.pdf Right Triangles and Trigonometry: Pythagorean Theorem https://www.khanacademy.org/math/geometry/right_triangles_topic/pyth_theor/v/pythagorean-theorem Pythagorean Theorem https://www.ixl.com/math/grade-8/pythagorean-theorem-word-problems Pythagorean Theorem http://regentsprep.org/Regents/math/ALGEBRA/AT1/indexAT1.htm

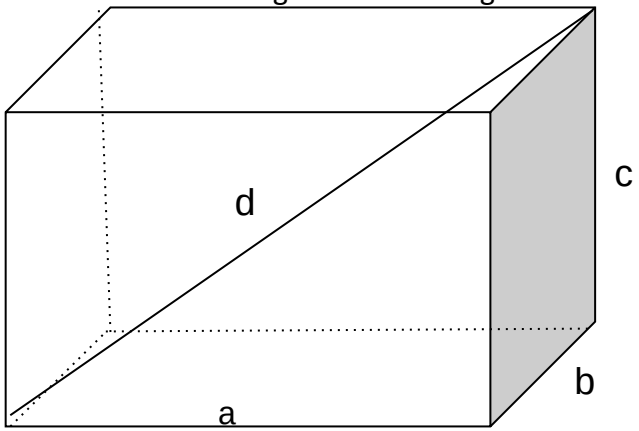
Sample Formative Assessment Tasks/Questions
8.GM.6: The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not? *Answer* Yes, the three towns form a right triangle. If a = 180, b = 240, and c = 300, then a^2 = 32,400, b^2 = 57,600, and c^2 = 90,000. $a^2 + b^2 = c^2$ 32400 + 57600 = 90000

8.GM.7: The Irrational Club wants to build a treehouse. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the treehouse be off the ground?

Answer
$a^2 + b^2 = c^2$
$a^2 + 5^2 = 9^2$
$a^2 + 25 = 81$
$a^2 = 56$
$a \approx 7.5$

Therefore, the treehouse will be approximately 7.5 feet off the ground.

8.GM.7: Find the length of *d* in the figure below if *a* = 8 in., *b* = 3 in., and *c* = 4 in.

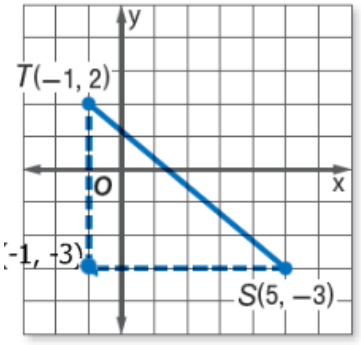


Answer
$a^2 + b^2 + c^2 = d^2$
$8^2 + 3^2 + 4^2 = d^2$
$64 + 9 + 16 = d^2$
$89 = d^2$
$d \approx 9.43$ inches

8.GM.8: Points *A*(3, 8) and *B*(*x*, *y*) are separated by a distance of $\sqrt{45}$ units. Give the coordinates of two possible locations for point B on a coordinate plane.

Sample Answers
B(6, 2)
B(9, 5)

8.GM.7, 8.GM.8: Use the diagram to the below , ΔSTU, to answer the following questions.



a. What is the length of TU? _____

b. What is the length of SU? _____

c. What is the length of ST?

Exact answer_____ Approximate answer _____

Answer

a. 5 units
b. 6 units
c. Exact answer: $\sqrt{61}$units; Approximate answer: 7.8 units

8.GM.9: A conical glass flower vase has a base that is 6 inches in diameter and the vase holds approximately 135 cubic inches of water. What is the height of the vase? Exact answer_____ Approximate answer _____

Answer

The height of the vase is $\frac{15}{\pi}$ feet, which is approximately 4.8 feet.

8.GM.9: A wax candle is in the shape of a right circular cone. The height of the candle is 9 cm. and the candle contains approximately 167.55 $cm.^3$ of wax. What is the radius of the candle?

Answer
The radius of the candle is 4.2cm.

Source: LearnZillion

8.GM.9, 8.EEI.2: Circleville has a spherical landmark with a volume of 2304π cubic feet. Cone Town would like to build a comparable structure with the same volume. If the town council determines that the new cone-shaped landmark must be 48 feet tall, what is the expected circumference of the structure's base?

Answer
Students must calculate the radius of the sphere (12 feet) and use that information to determine the radius of the cone is 12 feet as well. With a radius of 12ft, the circumference of the cone's base will be 24πfeet.

8.GM.9, 8.EEI.2: What is the side length of a cube with an area of 64 ft^3?

Answer
The side length would be 4 ft.

8.GM.9, 8.EEI.2: If $x^3 = \frac{1}{8}$, what is x?

Answer

$\sqrt[3]{x} = \sqrt[3]{\frac{1}{8}}$

$x = \frac{1}{2}$

Unit Title
Functions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.F.1 Explore the concept of functions. a. Understand that a function assigns to each input exactly one output. b. Relate inputs (*x*-values or domain) and outputs (*y*-values or range) to independent and dependent variables. c. Translate among the multiple representations of a function, including mappings, tables, graphs, equations, and verbal descriptions. d. Determine if a relation is a function using multiple representations, including mappings, tables, graphs, equations, and verbal descriptions. e. Graph a function from a table of values. Understand that the graph and table both represent a set of ordered pairs of that function. ○ A function is a specific type of relationship in which each input has a unique output. ○ The domain is every value *x* can be, and range is every value *y* can be. ○ The domain represents independent values, and the range represents dependent values. ○ The Vertical Line Test can be used to identify functions represented as graphs. ○ Define function ○ Identify the domain and range of a relation ○ Determine if a graph is a function ○ Determine if a set of points is a function ○ Identify functions from an equation ○ Calculate the *y*-value for an equation when given the *x*-value ○ Calculate the *x*-value for an equation when given the *y*-value ○ Create a table for an equation ○ Determine if a table is a function ○ Determine if an equation is a function by looking at it ○ Represent a function in the form of ordered pairs, mapping, graph, or listing

8.F.2 Compare multiple representations of two functions, including mappings, tables, graphs, equations, and verbal descriptions, in order to draw conclusions.

- Find the slope of a line from a graph
- Find the slope of a line from a table
- Find the slope of a line from an equation
- Compare and explain slopes [unit rate]
- Identify properties of a function
- Compare/contrast two functions using the same representation (graphically, numerically, verbally)
- Compare/contrast two functions using different representations
- Compare functions represented in different forms to determine which has the greater rate of change (slope)

New Academic Vocabulary for This Unit

- Domain/ Input
- Mappings
- Range/Output
- Function

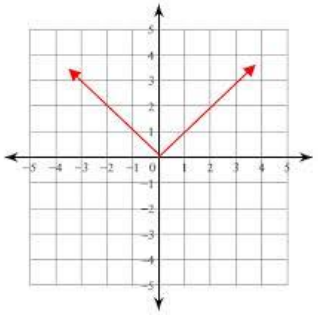
Prior Knowledge Required for this Unit
• Investigate the relationships between two numerical patterns (5.ATO.3) • Extend the concept of numerical patterns and expressions to algebraic expressions (6.EEI.2) • Graph ordered pairs on a coordinate grid (6.NS.6) • Create table and graph equivalent ratios to real world situation on coordinate grid (6.RP.3) • Write and solve one step linear equations (6.EEI.7) • Solve multi step linear equations (7.EEI.4) • Solve multi step linear equations with rational coefficients (8.EEI.7)
Subsequent Knowledge Related to this Unit
In subsequent Grade 8 units, students will use their understanding of functions to work with linear functions. Students will continue to build on these concepts through high school mathematics courses, where they will cover a variety of functions (e.g. absolute value, exponential, quadratic, radical, rational) and identify key features of the graph (e.g. maximums/minimums, intercepts, and asymptotes). Students will also explore the concept of a function's inverse and determine whether it is a function.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the multiple representations of functions.

Potential Instructional Strategies/Lessons

- **Vertical Line Test**

You can determine whether a graph is or is not a function by using the vertical line test.

Example
Does the graph show a function?



Answer
Yes, this graph does represent a function because it passes the vertical line test. This means for each input value, *x*, there is only one associated output value, *y*.

Resources
8.F.1 - This site provides an introductory example for recognizing relationships characterized as functions. Students analyze both tabular and graphical representations. https://www.illustrativemathematics.org/content-standards/8/F/A/tasks/1928 8.F.1 - This site allows students to practice, using an interactive game, writing a rule to a function table http://www.mathplayground.com/functionmachine.html 8.F.1 - This site has many videos on function tables and information pertaining to functions. http://www.watchknowlearn.org/Category.aspx?CategoryID=4708 8.F.1 - This site allows students to enter an input and figure out the rule. https://www.ixl.com/math/grade-4/input-output-tables-with-addition-subtraction-multiplication-and-division

Sample Formative Assessment Tasks/Questions

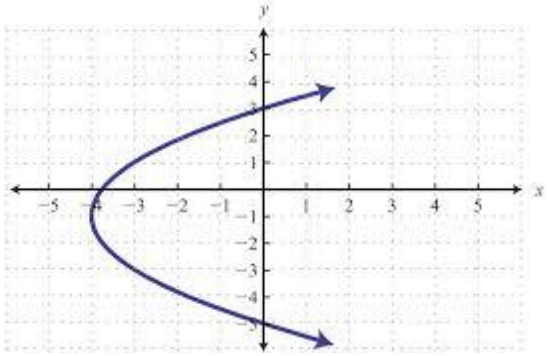
8.F.1: Use the data in input/output table to determine the function rule.

Let x = input
Let y = output

Input	Output
2	3
0	-3
-2	-9
-1	-6
5	12
100	297
10	27

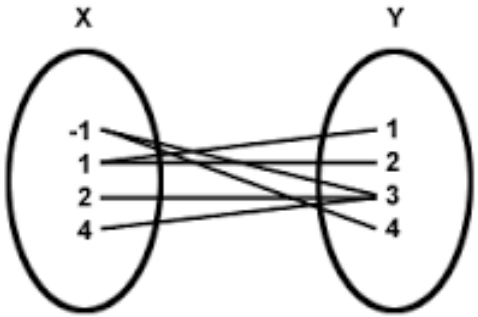
Answer
$y = 3x - 3$

8.F.1: Does the following graph below represent a function? Justify your reasoning.



Answer
No, the graph does not represent a function. This graph does not pass the vertical line test.

8.F.1: Look at the mapping below, does it represent a function? Justify your reasoning.



Answer
No, this mapping does not represent a function because for inputs -1 and 1, there is more than one output.

8.F.1: Does the following table represent a function? Justify your reasoning.

x	*y*
-1	3
0	0
1	-3
2	-6

Answer
Yes, the table does represent a function because for each input (x), there is exactly one output (y).

8.F.1: Complete the following function table.

x	$y = 2x - 8$	*y*
-2		
-1		
0		
1		

Answer

x	$y = 2x - 8$	*y*
-2	2(-2) - 8	-12
-1	2(-1) - 8	-10
0	2(0) - 8	-8
1	2(1) - 8	-6

8.F.2: Devon and Tara each received Moe's gift cards for their birthdays. Tara eats at Moe's every Monday, and the balance on her gift card can be modeled by the equation: $y = 20 - 4x$, where y is the gift card balance and x is the number of weeks she has used the card. Devon's balance is represented by the table below:

Weeks	0	2	4	6
Balance	30	21	12	3

a. Which person, Devon or Tara spends more money at Moe's each week?
b. At these rates, whose gift card would you expect to last longer? Explain your answer.

Answer

a. Tara spends $4 each week, and Devon spends $4.50 each week. Devon spends more money at Moe's each week.
b. Even though Devon spends more money each week, his gift card will last longer. Tara's gift card will have a zero balance after 5 weeks, but Devon will be able to use his card for 6 full weeks and still have a balance of $3.00 remaining.

Unit Title
Linear Functions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.EEI.5 Apply concepts of proportional relationships to real-world and mathematical situations. a. Graph proportional relationships. b. Interpret unit rate as the slope of the graph. c. Compare two different proportional relationships given multiple representations, including tables, graphs, equations, diagrams, and verbal descriptions. ○ Graph a proportional relationship given a table, equation or contextual situation. ○ Recognize unit rate as slope and interpret the meaning of the slope in context. ○ Understand that proportional relationships will contain the point (0, 0). ○ Compare different representations of two proportional relationships represented as real-world situations, graphs, or equations. **8.EEI.6** Apply concepts of slope and y-intercept to graphs, equations, and proportional relationships. a. Explain why the slope, m, is the same between any two distinct points on a nonvertical line using similar triangles. b. Derive the slope-intercept form ($y = mx + b$) for a non-vertical line. c. Relate equations for proportional relationships ($y = kx$) with the slope-intercept form ($y = mx + b$) where $b = 0$. ○ Understand that the Constant of Proportionality is the same as the unit rate and the slope of any linear function. ○ Determine the slope of a line as the ratio of the leg lengths of similar right triangles. ○ Explain why the slope is the same between any two distinct points on a line using similar right triangles. ○ Write an equation in the form $y = mx + b$ from a graph of a line on the coordinate plane. **8.F.3** Investigate the differences between linear and nonlinear functions using multiple representations (i.e., tables, graphs, equations, and verbal descriptions). a. Define an equation in slope-intercept form ($y = mx + b$) as being a linear function. b. Recognize that the graph of a linear function has a constant rate of change. c. Provide examples of nonlinear functions. ○ Understand that nonlinear functions do not have a constant rate of change ○ Distinguish between linear and non-linear functions given their representation (i.e., tables, graphs, equations, and verbal descriptions). ○ Recognize functions written in the form $y = mx + b$ are linear and that every linear function can be written in the form $y = mx + b$.

8.F.4 Apply the concepts of linear functions to real-world and mathematical situations.
a. Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$.
b. Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.
c. Construct a function in slope-intercept form that models a linear relationship between two quantities.
d. Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.
e. Explore the relationship between linear functions and arithmetic sequences.

- Recognize that the y-intercept is the point at which the graph crosses the y-axis.
- Recognize that every y-intercept will have an *x* value of zero.
- Determine and interpret the initial value and rate of change given two points, table, graph, equations, and verbal description of a linear relationship.
- Write the equation of a line given two points, table, graph, equations, and verbal description of a linear relationship.
- Recognize that the common difference of an arithmetic sequence is related to the slope of a linear function.

8.F.5 Apply the concepts of linear and nonlinear functions to graphs in real-world and mathematical situations.
a. Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous).
b. Sketch the graph of a function from a verbal description.
c. Write a verbal description from the graph of a function with and without scales.

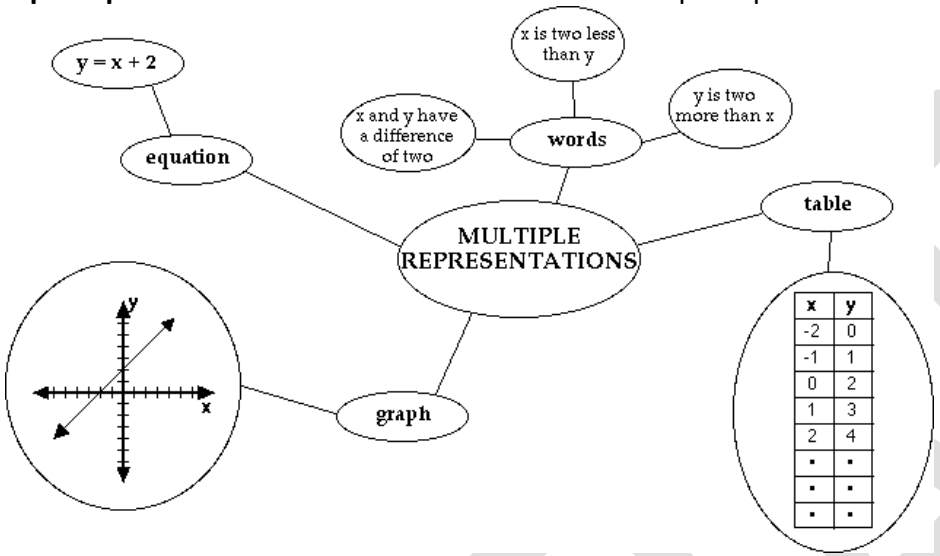
- Recognize that graphs should be read from left to right
- Recognize that a function that is constant will be represented with a horizontal line
- Recognize that a graph is increasing if the y-values increase as the x-values increase
- Recognize that a graph is decreasing if the y-values decrease as the x-values increase
- Recognize that discrete data is “countable” and continuous data is “measurable”
- Recognize that if a graph contains discrete data the points will not be connected with a line
- Recognize that if a graph contains continuous data the points will be connected with a line
- Recognize that linear functions will not have a maximum or minimum value
- Create a graphical representation given the description of the relationship between two quantities

New Academic Vocabulary for This Unit
• Slope-Intercept Form • Discrete • Continuous • Y-intercept • Domain • Range

Prior Knowledge Required for this Unit
• Number and shape patterns (sequences) (5.ATO.3) • Plot values in all quadrants of the coordinate plane (6.NS.6) • Understand and calculate unit rates (6.RP.2) • Recognize similar triangles and know that they have proportional sides. (8.GM.4, 8.GM.5)
Subsequent Knowledge Related to this Unit
In high school mathematics classes, students will expand their understanding of proportional relationships/direct variation to inverse variation ($y=\frac{k}{x}$). Knowledge of linear and nonlinear functions will be extended to include absolute value, cubic, exponential, quadratic, radical, rational, and trigonometric functions. Students will also explore relationships that are not defined as functions including circles, ellipses, hyperbolas, and horizontal parabolas ($x=y^2$). Additionally, students will explore geometric sequences that model exponential relationships with terms sharing a common ratio.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the multiple representations of linear and nonlinear functions.

Potential Instructional Strategies/Lessons

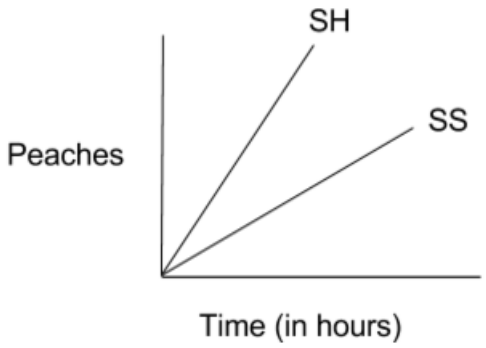
- **Multiple representations** – Provide students with multiple representations of functions.



Source: Florida Center for Instructional Technology

- **Real-world Connections**
- **Slope Triangles** - This lesson offers students a method for finding the slope of a line from its graph.
  - https://illuminations.nctm.org/Lesson.aspx?id=2570

Resources
8.EEI.5, 8.EEI.6, 8.F.3 - This unit contains lessons and tasks aligned to this unit. https://www.georgiastandards.org/Georgia-Standards/Frameworks/8th-Math-Unit-5.pdf 8.F.3 - This site walks through how to Investigate the differences between linear and nonlinear functions using multiple representations, also provides examples for students to review. This site is mainly for the teacher to use to help students understand the lesson of linear versus nonlinear relationships, and provides a great lesson. http://www.cpalms.org/Public/PreviewResourceLesson/Preview/48283 8.F.4 - This site introduces slope as a constant rate of change and provides real world examples (including the graphs that go along with each example). http://www.regentsprep.org/regents/math/algebra/ac1/rate.htm 8.F.4 - This site host a great game for students to practice recognizing the slope and y-intercept of a line. http://hotmath.com/hotmath_help/games/kp/kp_hotmath_sound.swf 8.F.4 - This site provides a complete study of linear functions and relationships, as it teaches and allows interactive activities to check for understanding of slope and y-intercept. http://www.watertown.k12.ma.us/wms/math/math_help/gradeeight/moving/msa.html 8.F.4 - The lesson is based upon Aesop's fable, "The Crow and the Pitcher," and involves students making predictions and conducting experiments to determine how many pebbles the crow would need to add to the pitcher in order to bring the water to drinking height. http://illuminations.nctm.org/lesson.aspx?id=3667 8.F.5 - This site provides detailed real life examples of linear and nonlinear functions in the real world. https://www.engageny.org/sites/default/files/resource/attachments/math-g8-m5-teacher-materials.pdf

Sample Formative Assessment Tasks/Questions
8.EEI.5: The graph below shows the number of peaches sold by Sunny Slope (SS) and Strawberry Hill (SH) Farms over a period of several hours. Sunny Slope sold 35 peaches per hour.    Write an equation to describe the number of peaches sold by Strawberry Hill Farm after, *h*, hours. *Answer* Student answers will vary. Sample answers may include 50 peaches per hour, 58 peaches per hour, or 63 peaches per hour.
8.EEI.6: Ishmael is working as a math tutor. He charges $30 per hour to tutor middle school students. If Ishmael has to travel to a student's home, there is a fixed charge of $5. a. Write equations to represent the cost of tutoring, *x*, hours both with and without transportation charges. b. What do the equations' values mean in the context of the graph? *Answer* a. Without Transportation: $y = 30x$ and With Transportation: $y = 30x + 5$. b. The cost of $30 per hour represents the slope of the lines, and the additional charge of $5 for travel represents the y-intercept for the equation with the additional charge. Since the equation for the cost without a travel charge is $y = 30x$, the value of the y-intercept is zero.

8.F.3a: Given the three equations, $y = \frac{1}{x} - 4$, $y = \frac{1}{2}x + 3$, and $y = x^2 + 1$, one represents a linear function and the others do not. Determine which function is linear and explain why that function is linear.

Answer

The equation that is a linear function is $y = \frac{1}{2}x + 3$. It is the only equation that has exactly one *x*-value for each *y*-value. Additionally, the values will increase by the same amount.

8.F.3b: Given the following situations, determine which one would represent a linear function. Once identified, determine the constant rate of change of the linear function.

a. $16 = x^2 + y^2$

b. The path of a football being thrown downfield and landing on the ground

c. An equation where the cost earned babysitting is $5 for each hour

d.

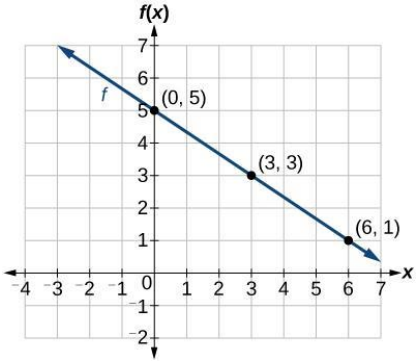
x	Y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Answer

Situation is c is a linear function because it would have exactly one *x*-value for each *y*-value. Additionally, the values will increase by the same amount.

8.F.3b:

a. Does the graph below represent a linear function? If so, how do you know?



b. What is the constant rate of change?

Answer

a. The graph below is a linear function because it has exactly one *x*-value for each *y*-value. Additionally, the values decrease by the same amount.

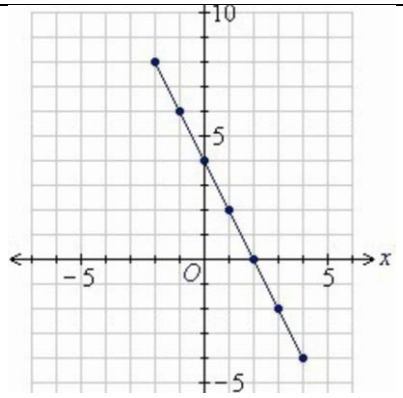
b. The constant rate of change would be the slope of the line. For this graph, the constant rate of change is $-\frac{2}{3}$.

8.F.3c: Give an example of a nonlinear function in two different representations of either a verbal description, an equation, a table of values, etc. Justify why your representations are nonlinear.

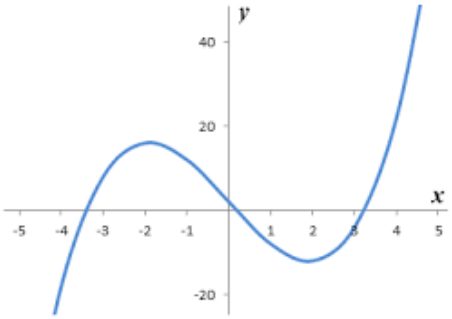
Answers will vary.

8.F.3c: For each example, write true or false, to indicate whether the graph represent a linear function.

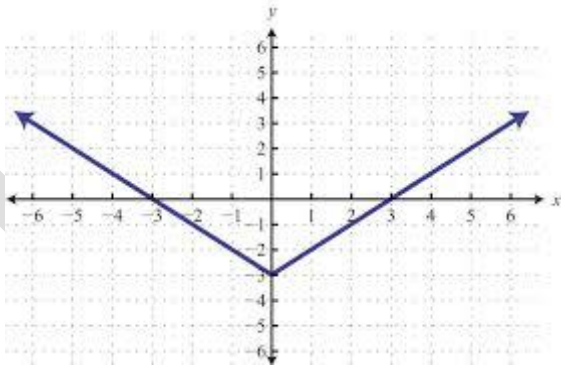
a.



b.



c.



Answer
a. True
b. False
c. False

8.F.4a: Using the table below, determine the equation, slope, and the *y*-intercept of the function.

x	***y***
1	6
2	10
3	14
4	18

Answer
The equation that represents this function is y = 4x + 2. The slope of the function is 4 and the y-intercept is 2.

8.F.4b: Which of the following is the equation of the linear function containing the points (0, 3) and (-2, 5)?

a. $y = x + 3$
b. $y = -4x + 5$
c. $y = 4x + 3$
d. $y = -x + 3$

Answer
Since the problem contains the points (0, 3), 3 is the initial value, or y-intercept, of this equation. Using the two points to calculate slope, you find the slope is -1. Therefore, the equation of this function is d. $y = -x + 3$.

8.F.4c,d: Nick Jonas has joined an online movie rental club for a month while he is on vacation in Fiji. The rental club charges a $7.00 fee for the month-long membership, plus $0.30 for each movie rented. Write a function to represent this situation. Interpret the meaning of the slope and y-intercept of your equation in this context.

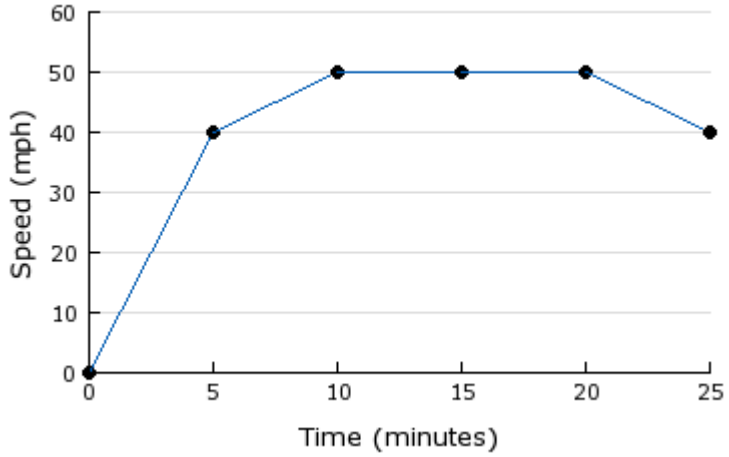
Answer
In this situation, $7.00 represents the initial fee, the y-intercept, for the month rental. The rate for renting each movie is $0.30 per movie, which can be represented as a fraction $\frac{3}{10}$ or the decimal 0.3, and represents the slope. If we let *y* represent the total cost for a month and let *x* represent the number of movies rented in that month, then the equation $y = \frac{3}{10}x + 7$ or $y = 0.3x + 7$ can be used to represent this function.

8.F.4e: Use the following sequences of numbers to answer the questions below.
Sequence A: 10, 20, 30, 40, 50 , ...
Sequence B: 2, 4, 8, 16, 32, ...
Sequence C: 4, 6, 8, 10, 12, ...

Which of these sequences could be written as a linear function? Why?

Answer
Sequences A and C can be written as linear functions. Both sequences increase at a constant rate of change. Sequence A increases by 10 and the linear equation y = 10x can be used to find any term in the sequence where x is the term number and y is the term in the sequence. Likewise, sequence C increases by 2 and the linear equation y = 2x + 2 can be used to find any term in the sequence where x is the term number and y is the term in the sequence. Sequence B cannot be written as a linear function because it does not increase at a constant rate.

8.F.5: The graph below represents a period of time that Jason was riding his bike on a recent trip.



a. Describe the interval in which the graph exhibits the following attributes and justify your answer.
   Constant:
   Increasing:
   Decreasing:

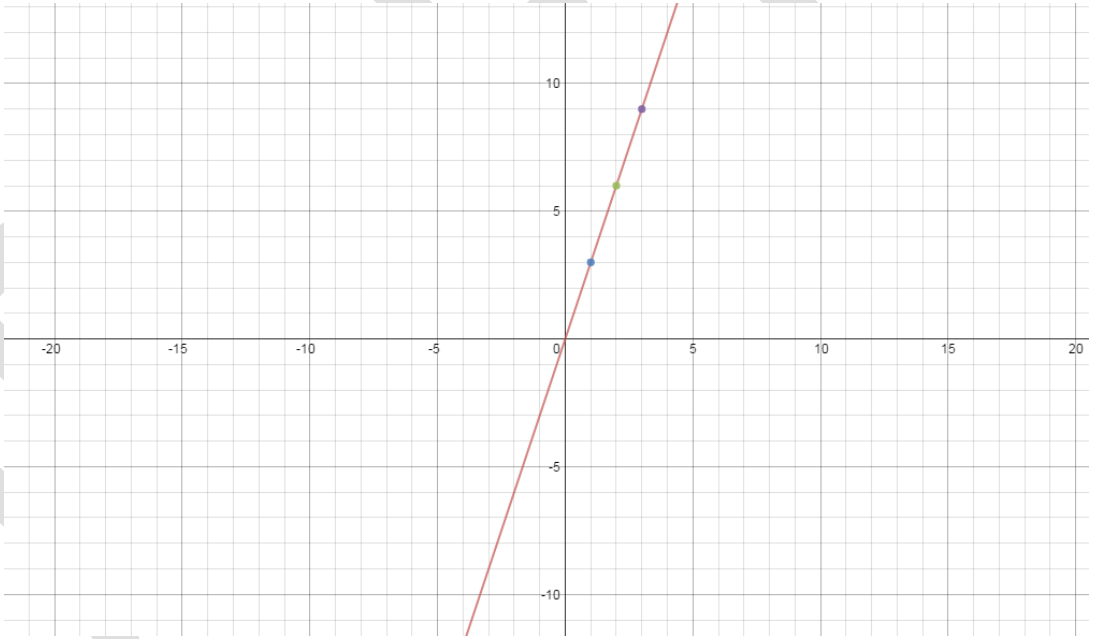
b. Is this graph a display or a linear or nonlinear function? Explain.

Answer
Constant: the graph is constant between the time interval of 10 minutes and 20 minutes. The reason why is because as the x-value increases the y-value remains the same value. Increasing: the graph is increasing between the time intervals of 0 minutes and 10 minutes. The reason why is because as the x-value increases the y-value is also increasing. Decreasing: the graph is decreasing between the time interval of 20 minutes and 25 minutes. The reason why is because as the x-value increases the y-value is decreasing.
The graph represents a nonlinear function because throughout the domain and range of this graph there is not a constant rate of change throughout the graph.

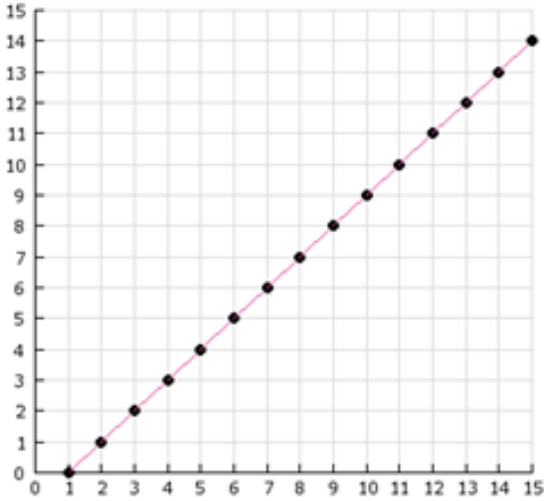
8.F.5b: Sketch a graph that describes the situation below:
Annie is 3 times as old as Sandy

Answer
In order to graph the description you would first need to determine the independent (x) and dependent (y) variables for the situation. After you determine that Annie's age depends on the age of Sandy you can then assign at least three different values to determine your ordered pairs to graph on the coordinate plane.

x (Sandy)	y (Annie)
1	3
2	6
3	9



8.F.5c: Using the graph below write a description that would describe the graph.



Answer (answers will vary)
After sitting at the traffic light for a minute, I traveled I went a distance of 1 mile every minute.

Unit Title
Statistics with Linear Models
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.DSP.1 Investigate bivariate data. a. Collect bivariate data. b. Graph the bivariate data on a scatter plot. c. Describe patterns observed on a scatter plot, including clustering, outliers, and association (positive, negative, no correlation, linear, nonlinear). ○ Recognize that bivariate data consists of two variables. ○ Understand that bivariate data can be graphed on the coordinate plane. ○ Collect, record, and construct a set of bivariate data using a scatter plot. ○ Determine whether the relationship between bivariate data is approximately linear or nonlinear by examination of a scatter plot. ○ Interpret patterns on a scatter plot such as clustering, outliers, and positive, negative, or no association. **8.DSP.2** Draw an approximate line of best fit on a scatter plot that appears to have a linear association and informally assess the fit of the line to the data points. ○ Recognize that straight lines can be used on scatter plots to model the relationship between two quantitative variables. ○ Place a straight line on a scatter plot that closely fits the data points. ○ Judge how well the trend line fits the data by looking at the closeness of the data points. ○ Recognize there should be an equal number of data points above and below the trend line. **8.DSP.3** Apply concepts of an approximate line of best fit in real-world situations. a. Find an approximate equation for the line of best fit using two appropriate data points. b. Interpret the slope and intercept. c. Solve problems using the equation. ○ Use the equation of a linear model to solve problems. ○ Interpret the meaning of the slope as a rate of change and the meaning of the y-intercept in context given bivariate data.

8.DSP.4* Investigate bivariate categorical data in two-way tables.
a. Organize bivariate categorical data in a two-way table.
b. Interpret data in two-way tables using relative frequencies.
c. Explore patterns of possible association between the two categorical variables.
- Recognize that data is organized by rows and columns in two-way tables.
- Recognize relative frequency as the number of desired outcomes divided by all outcomes.
- Construct a two-way frequency table of categorical data.
- Interpret and describe relative frequencies for possible associations from a two-way table.

8.F.3 Investigate the differences between linear and nonlinear functions using multiple representations (i.e., tables, graphs, equations, and verbal descriptions).
a. Define an equation in slope-intercept form ($y = mx + b$) as being a linear function.
b. Recognize that the graph of a linear function has a constant rate of change.
c. Provide examples of nonlinear functions.
- Understand that nonlinear functions do not have a constant rate of change
- Distinguish between linear and non-linear functions given their representation (i.e., tables, graphs, equations, and verbal descriptions).
- Recognize functions written in the form $y = mx + b$ are linear and that every linear function can be written in the form $y = mx + b$.

8.F.4 Apply the concepts of linear functions to real-world and mathematical situations.
a. Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$.
b. Determine the slope and the y-intercept of a linear function given multiple representations, including two points, tables, graphs, equations, and verbal descriptions.
c. Construct a function in slope-intercept form that models a linear relationship between two quantities.
d. Interpret the meaning of the slope and the y-intercept of a linear function in the context of the situation.
e. Explore the relationship between linear functions and arithmetic sequences.
- Recognize that the y-intercept is the point at which the graph crosses the y-axis.
- Recognize that every y-intercept will have an *x* value of zero.
- Determine and interpret the initial value and rate of change given two points, table, graph, equations, and verbal description of a linear relationship.
- Write the equation of a line given two points, table, graph, equations, and verbal description of a linear relationship.
- Recognize that the common difference of an arithmetic sequence is related to the slope of a linear function.

8.F.5 Apply the concepts of linear and nonlinear functions to graphs in real-world and mathematical situations.
a. Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous).
b. Sketch the graph of a function from a verbal description.
c. Write a verbal description from the graph of a function with and without scales.

- Recognize that graphs should be read from left to right
- Recognize that a function that is constant will be represented with a horizontal line
- Recognize that a graph is increasing if the y-values increase as the x-values increase
- Recognize that a graph is decreasing if the y-values decrease as the x-values increase
- Recognize that discrete data is "countable" and continuous data is "measurable"
- Recognize that if a graph contains discrete data the points will not be connected with a line
- Recognize that if a graph contains continuous data the points will be connected with a line
- Recognize that linear functions will not have a maximum or minimum value
- Create a graphical representation given the description of the relationship between two quantities

New Academic Vocabulary for This Unit

- Bivariate Data
- Line of Best Fit
- Two-way Table
- Relative Frequency
- Scatter Plot

Prior Knowledge Required for this Unit

- Proportional reasoning (6.RP.1-3, 7.RP.1-3)

Subsequent Knowledge Related to this Unit

In high school math courses, students will begin to identify linear, quadratic, and exponential relationships among data values. Also, in high school math courses, students will determine the regression equations for linear, quadratic, and exponential relationships.

Relationship Among Standards in this Unit

Standards in this unit will extend understanding of relationships that exist among data collection and bivariate relationships. Concepts are also analyzed in association with linear and nonlinear relationships.

Potential Instructional Strategies/Lessons
• Manipulatives - Use manipulatives to approximate the line of best fit.   ○ Spaghetti noodles
Resources
8.DSP.1, 8.DSP.2, 8.DSP.3 - These activities allow students to actively engage in data collection and construction of the line of best fit. http://mathbits.com/MathBits/TINSection/Statistics1/lineBestFit.html http://illuminations.nctm.org/Lesson.aspx?id=2157 https://www.teachingchannel.org/videos/stem-lesson-ideas-bungee-jump http://www.sophia.org/packets/bivariate-data-two-variables--2 8.DSP.1 - This site introduces bivariate data, and takes a look at how to represent bivariate data in tables and on a scatter plot. https://www.mathsisfun.com/data/univariate-bivariate.html 8.DSP2, 8.DSP.3 - This site provides a lesson with integrated technology that allows students to explore the idea of line of best fit and assess their work through the use of a TI-Nspire calculator. http://mathbits.com/MathBits/TINSection/Statistics1/lineBestFit.html 8.DSP.4 - This site provides an introductory slide show, guided practice activities, assessment, and answer keys for two-way tables. http://www.cpalms.org/Public/PreviewResourceLesson/Preview/65965

8.F.3 - This site walks through how to Investigate the differences between linear and nonlinear functions using multiple representations, also provides examples for students to review. This site is mainly for the teacher to use to help students understand the lesson of linear versus nonlinear relationships, and provides a great lesson.
http://www.cpalms.org/Public/PreviewResourceLesson/Preview/48283

8.F.4 - This site introduces slope as a constant rate of change and provides real world examples (including the graphs that go along with each example).
http://www.regentsprep.org/regents/math/algebra/ac1/rate.htm

8.F.4 - This site host a great game for students to practice recognizing the slope and y-intercept of a line.
http://hotmath.com/hotmath_help/games/kp/kp_hotmath_sound.swf

8.F.4 - This site provides a complete study of linear functions and relationships, as it teaches and allows interactive activities to check for understanding of slope and y-intercept.
http://www.watertown.k12.ma.us/wms/math/math_help/gradeeight/moving/msa.html

8.F.5 - This site provides detailed real life examples of linear and nonlinear functions in the real world.
https://www.engageny.org/sites/default/files/resource/attachments/math-g8-m5-teacher-materials.pdf

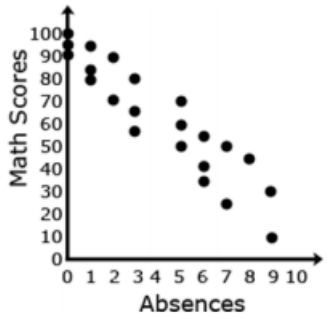
Sample Formative Assessment Tasks/Questions

8.DSP. 1, 2, 3: Use the chart below to complete parts a, b, c, d, and e.

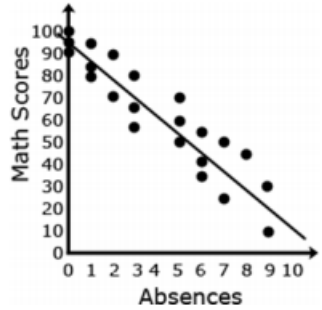
a. Given data from students' math scores and absences, make a scatterplot.
b. Draw a linear model paying attention to the closeness of the data points on either side of the line.
c. From the linear model, determine an approximate linear equation that models the given data.
d. Interpret the slope and y-intercept of the line in the context of the problem.
e. Use the equation to determine what a student with 4 absences should expect to receive as a math score. Then compare this value to the line.

Answer

a.



b.



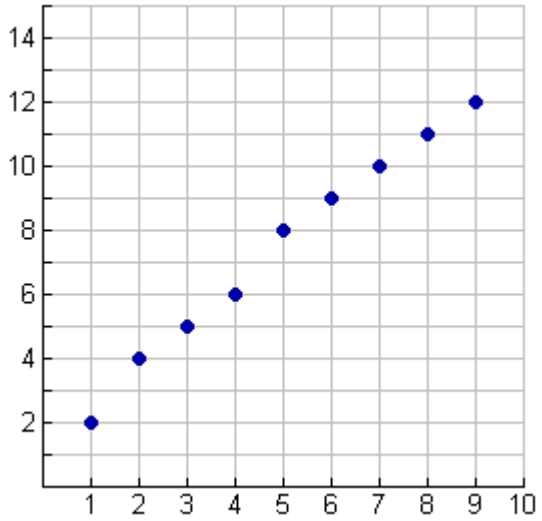
c. $y = -\frac{25}{3}x + 95$

d. In the equation, 95 represents the *y*-intercept and $-\frac{25}{3}$ represents the slope of the line.

e. A student with 4 absences should expect to receive a math score of about 62.

Source: Kansas Association of Teachers of Mathematics (KATM) Flipbook

8.DSP.2: You are asked to write an equation for the line of best fit for the scatter plot shown below without the use of a graphing calculator.



What should you do first?
a. Connect the points together.
b. Find the slope using (1,2) and (9,12).
c. Find the slope using (4,6) and (5,8).
d. Decide which two points give the most representative straight line.

Answer
d. Decide which two points give the most representative straight line.

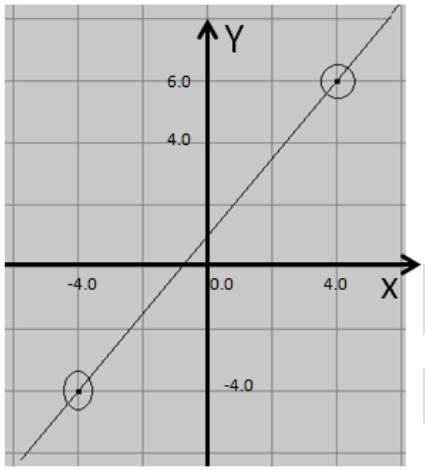
8.F.3: Categorize the following functions as linear or nonlinear.

a. $y(x) = -x^2 + 4$
b. $y(x) = 2x^3 + 1$
c. $y(x) = 2x - 1$
d. $y(x) = x - 1$

Answer

a. nonlinear
b. nonlinear
c. linear
d. linear

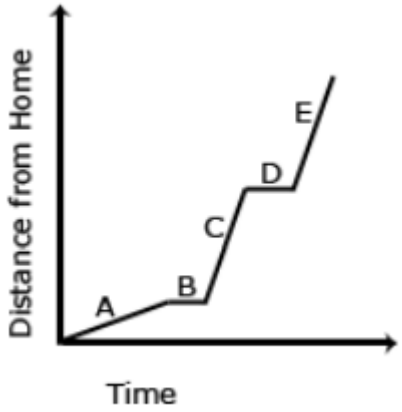
8.F.4: What is the slope for this line?



Answer

The slope of the line is $\frac{5}{4}$ or 1.25.

8.F.5: The graph below shows a John's trip to school. He walks to his Sam's house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story.

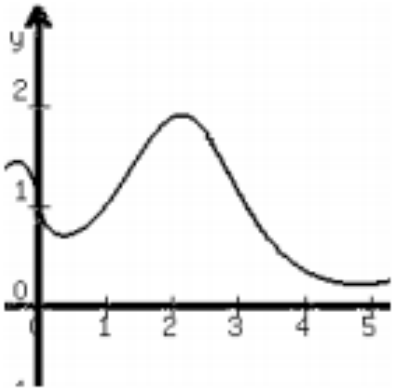


Answer

A. John is walking to Sam's house at a constant rate.
B. John gets to Sam's house and is waiting for the bus.
C. John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John's walking rate.
D. The bus stops.
E. The bus resumes at the same rate as in part C.

Source: NC DPI 8th Grade Mathematics Unpacked Contents

8.F.5: Describe the graph of the function between $x = 2$ and $x = 5$?



Answer
The graph is non-linear and decreasing.

Source: NC DPI 8th Grade Mathematics Unpacked Contents

Unit Title
Systems of Equations
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
8.EEI.7 Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations. a. Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides. b. Recognize the three types of solutions to linear equations: one solution ($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$). c. Generate linear equations with the three types of solutions. d. Justify why linear equations have a specific type of solution. ○ Recognize when an equation results in the form a=a (where a is a numerical value, example: 9=9), then the equation will have infinitely many solutions because a will always be equal itself. ○ Recognize when an equation results in the form a=b (where a and b are distinct numerical values, example 9=10), then the equation will have no solutions because the two values will never be equivalent to one another. ○ Create equations with no solution, infinite solutions, and one solution. **8.EEI.8** Investigate and solve real-world and mathematical problems involving systems of linear equations in two variables with integer coefficients and solutions. a. Graph systems of linear equations and estimate their point of intersection. b. Understand and verify that a solution to a system of linear equations is represented on a graph as the point of intersection of the two lines. c. Solve systems of linear equations algebraically, including methods of substitution and elimination, or through inspection. d. Understand that systems of linear equations can have one solution, no solution, or infinitely many solutions. ○ Recognize that intersecting lines will have one solution. ○ Recognize that parallel lines will have no solution because the two lines do not intersect. ○ Recognize that coinciding lines will have infinitely many solutions because they are the same line.

New Academic Vocabulary for This Unit
• No solution • Infinitely many solutions • Systems of Equations (simultaneous equations) • Substitution method • Elimination method (Linear combination) • Coinciding Lines (coincident lines) • Point of intersection

Prior Knowledge Required for this Unit
• Infinite solutions (6.EEI.8) • Linear equations (6.EEI.7, 7.EEI.4) • Integer operations (7.NS.1, 7.NS.2) • Writing algebraic equations for mathematical situations (6.EEI.7, 6.EEI.8) • Convert equations to the slope-intercept form (8.EEI.6) • Graph equations and inequalities (6.EEI.8, 7.EEI.4)
Subsequent Knowledge Related to this Unit
In high school math courses, students will extend their understanding of linear equations to nonlinear equations (including but not limited to quadratic and exponential equations). Students will also extend understanding of systems of equations to include both linear and nonlinear relationships in addition to equations involving three variables.
Relationship Among Standards in this Unit
Standards in this unit explore the relationships that exist among the solutions for equations of lines and systems of equations.

Potential Instructional Strategies/Lessons

• Technology to graph systems of equations and identify points of intersection
  ○ Graphing Calculators
  ○ Desmos.com

Resources

8.EEI.7 - This site includes a sample performance task for the standard in addition to a rubric for assessing students' work.
http://www.d11.org/Instruction/Mathematics/6-8/Documents/BigIdeasMath_Public/Tasks_Gr08/8.EE.7b_Kennel_Fees_Task_BIM12.pdf

8.EEI.7 - This interactive site provides questions, assesses students' answers, and provides additional examples to strengthen student understanding.
https://www.buzzmath.com/badges/criteria/content-cc8-expressions-gold

8.EEI.8 - This site provides a sample performance task, rubric, and sample student solutions regarding systems of linear equations.
http://www.insidemathematics.org/assets/common-core-math-tasks/picking%20apples.pdf

Sample Formative Assessment Tasks/Questions

8.EEI.7: Solve each equation algebraically. Then, let each side of the equation equal *y* and graph. Use the graph to verify your solution that you found algebraically.

Equation	No Solution	One Solution	Infinitely Many Solutions
A. $7x + 21 = 21$			
B. $12x + 15 = 12x - 15$			
C. $-5x - 25 = 5x + 25$			

A. One solution.
B. No solution. (Students may think there is no difference between adding 15 on the left-hand side and subtracting 15 on the right-hand side.)
C. One solution. (Students may think there are infinitely many solutions because the left-hand side is the negative of the right-hand side.)

Source: Kansas Association of Teachers of Mathematics (KATM) Flipbook

8.EEI.7: Three students solved the equation $3(5x - 14) = 18$ in different ways, but each student arrived at the correct answer. Select all of the solutions that show a correct method for solving the equation.

a.

$$\begin{aligned} 3(5x-14) &= 18 \\ 8x-14 &= 18 \\ +14 &\quad +14 \\ \frac{8x}{8} &= \frac{32}{8} \\ x &= 4 \end{aligned}$$

b.

$$\begin{aligned} \frac{1}{3}\cdot 3(5x-14) &= 18\cdot\frac{1}{3} \\ 5x-14 &= 6 \\ +14 &\quad +14 \\ \frac{5x}{5} &= \frac{20}{5} \\ x &= 4 \end{aligned}$$

c.

$$\begin{aligned} 3(5x-14) &= 18 \\ \frac{15x}{15} - \frac{42}{15} &= \frac{18}{15} \\ +\frac{42}{15} &\quad +\frac{42}{15} \\ x &= \frac{60}{15} \\ x &= 4 \end{aligned}$$

Answer

a. This solution is the simplest to follow, but the method is incorrect.
b. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.
c. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.

Source: Kansas Association of Teachers of Mathematics (KATM) Flipbook

8.EEI.8: Plant A and Plant B are on different watering schedules. This affects their rate of growth.

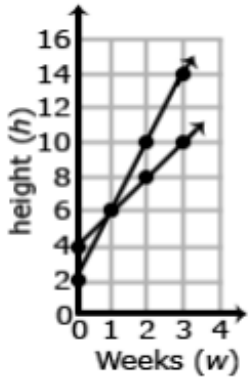
Plant A	
Week Number	Height (inches)
0	4
1	6
2	8
3	10

Plant B	
Week Number	Height (inches)
0	2
1	6
2	10
3	14

a. Given each set of coordinates, graph their corresponding lines.
b. Write an equation that represent the growth rate of Plant A and Plant B.
c. At which week will the plants have the same height?

Answer

a.



b. Plant A: $h = 2w + 4$ and Plant B: $h = 4w + 2$
c. After one week, the height of Plant A and Plant B are both 6 inches. The plants have the same height after one week.

Source: Kansas Association of Teachers of Mathematics (KATM) Flipbook